Opacity with Orwellian Observers and Intransitive Non-Interference

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Hiding and Disclosing Information
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Contents

ÉCOLE POLYTECHNIQUE MONTRÉAL

Outline

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Context

Need for relax security properties based on information flow

1. Quantify information flow
2. Declassify information

Opacity

Generic property expressing the capability of an observer to infer information from a secret behaviour of a partially observable system

Introduced in [Mazaré (WITS'2004), Bryans et al. (FAST'2005)]

In this talk:

Characterize opacity in the presence of intentional opacity disclosure

Opacity – Definition

Intuition

Environment has a perfect knowledge but a partial observation of the system's behavior.

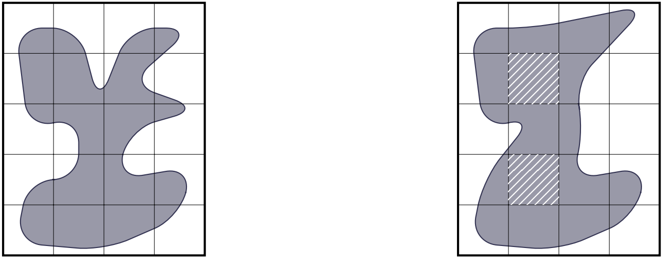
Definition

- $Exec(\mathcal{S})$: the set of executions of a system $\mathcal{S}$,
- $\mathcal{O} : Exec(\mathcal{S}) \rightarrow Obs^*$: an observation function.
- $\varphi \subseteq Exec(\mathcal{S})$: a predicate on $Exec(\mathcal{S})$ (the secret).

φ is opaque on $\mathcal{S}$ w.r.t. $\mathcal{O}$ if, for any $o \in Obs^*$, the following holds:

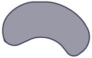
$$\mathcal{O}^{-1}(o) \not\subseteq \varphi.$$

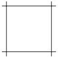
Opacity – Illustration



(a) φ is opaque on $\mathcal{S}$ w.r.t. $\mathcal{O}$

(b) φ is not opaque on $\mathcal{S}$ w.r.t. $\mathcal{O}$

 φ

 $\mathcal{O}^{-1}(o)$

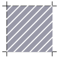
 Classes leaking their inclusion into φ

Figure: Opacity.

Computational power of observers

Definition

An observation function $\mathcal{O} : Exec(\mathcal{S}) \rightarrow Obs^*$ is called

- *static*: Observer's interpretation of events is fixed *a priori*
- *dynamic*: Observer's interpretation of the current event change dynamically depending on the the prefix of an execution observed so far
- *orwellian*: Observer's interpretation of the current event depends as well on the prefix as on the suffix of the execution

In this talk: Opacity w.r.t. a class of orwellian observers

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Deterministic transition systems

Definition (Labelled transition system)

A finite deterministic *labelled transition system* over Σ is a 4-tuple $G = (\Sigma, Q, \delta, q_0)$, where

- Σ: alphabet of events,
- Q: the state space,
- δ: a partial function from $Q \times \Sigma$ to Q (*labelled transition function*)
- q_0:*initial state*.

Some notations

- G is complete if δ is total
- G^q : G by starting from q
- $L(G) = \{s \in \Sigma^* : \delta(q_0, s)$ is defined$\}$ (*language of G*)
- $L_F(G) = \{s \in \Sigma^* : \delta(q_0, s) \in F\}$ (*language recognized* by $F \subseteq Q$)

Some LTS constructors

Definition (Synchronization product)

Let LTSs $G = (\Sigma, Q, \delta, q_0, F)$ and $G' = (\Sigma, Q', \delta', q_0', F')$. The *synchronization product* of G and G' denoted $G \times G'$ is the LTS $(\Sigma, Q \times Q', \delta \times \delta', (q_0, q_0'), F \times F')$ with

$$\delta \times \delta'((q, q'), a) = (\delta(q, a), \delta'(q', a))$$

Definition (Restriction)

Let $G = (\Sigma, Q, \delta, q_0, F)$, a LTS and $\Sigma' \subseteq \Sigma$. The restriction of G to Σ' denoted $G \setminus \Sigma'$ is the LTS $(\Sigma \setminus \Sigma', Q, \delta', q_0)$ with

$$\delta' = \delta|(\Sigma \setminus \Sigma')$$

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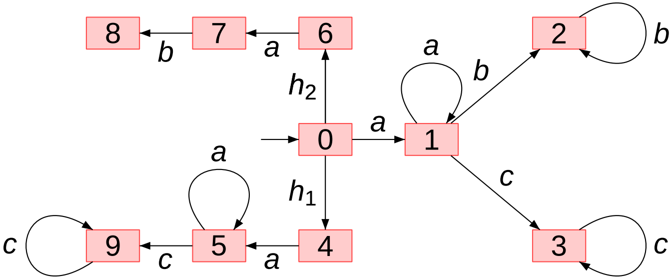
Opacity w.r.t. static observation

Natural projection

$$\pi_o(\epsilon) = \epsilon$$

$$\pi_o(s\cdot\alpha) = \begin{cases} \pi_o(s)\cdot\alpha & \text{if } \alpha \in \Sigma_o, \\ \pi_o(s) & \text{otherwise.} \end{cases}$$

- $\Sigma = \{h_1, h_2, a, b, c\}$
- $\Sigma_o = \{a, b, c\}$
- $\varphi = a^*(b^* + c^*)$



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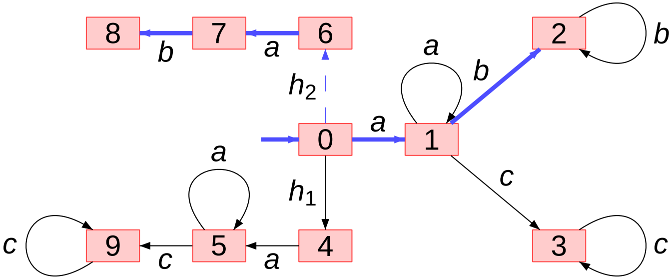

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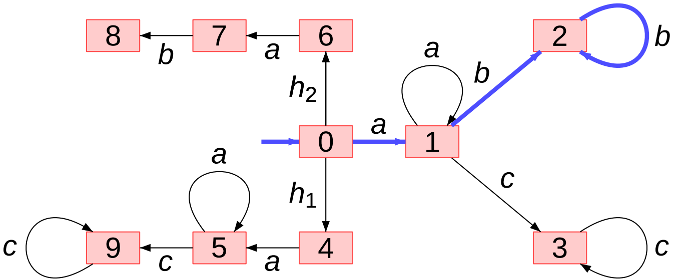

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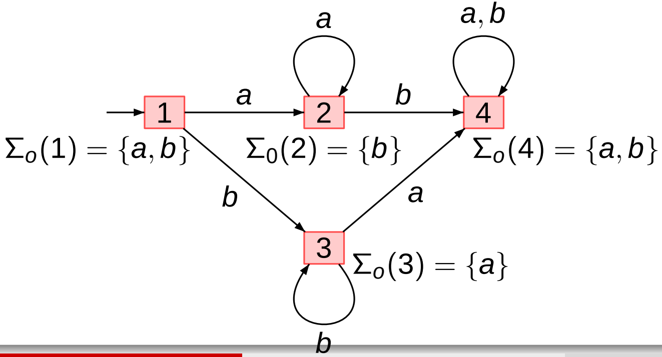
Opacity w.r.t. dynamic observation

Dynamic projection

$$\pi_{\mathcal{O}}(\epsilon) = \epsilon$$

$$\pi_{\mathcal{O}}(s \cdot \alpha) = \begin{cases} \pi_{\mathcal{O}}(s) \cdot \alpha & \text{if } \alpha \in \Sigma_o(\delta(q_o, s)), \\ \pi_{\mathcal{O}}(s) & \text{otherwise.} \end{cases}$$

- $\Sigma = \{a, b\}$
- $\mathcal{O} = (\Sigma, Q, \delta, q_0, \Sigma_0)$



Opacity verification problem

Opacity Verification Problem (OVP)

Is φ opaque on $\mathcal{S}$ w.r.t. $\mathcal{O}$?

Theorem (Bryans et al. (2005))

OVP is undecidable even for finite systems.

Theorem (Darondeau et al. (2007))

OVP is decidable for finite systems, regular secrets and static observation.

Theorem (Cassez et al. (2012))

OVP is decidable for finite systems, regular secrets and dynamic projections

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Theorem (Cassez et al. (2012))

OVP is decidable for finite systems, regular secrets and dynamic projections *Hint.:* $\pi(\varphi) \subseteq \pi(L \setminus \varphi)$ *is decidable for* π*: sequential*

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Orwellian projection

Orwellian projection

$$\pi_{o,d}(\epsilon) = \epsilon$$

$$\pi_{o,d}(s\alpha) = \begin{cases} s\alpha & \text{if } \alpha \in \Sigma_d, \\ \pi_{o,d}(s)\alpha & \text{if } \alpha \in \Sigma_o, \\ \pi_{o,d}(s) & \text{otherwise.} \end{cases}$$

- $\Sigma = \{h, d, l\}$
- $\Sigma_o = \{l\}$
- $\Sigma_d = \{d\}$
- $\varphi = \{hl\} \cup \{hdhl\}\{l\}^*$

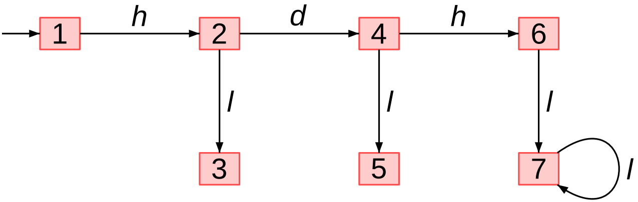

Figure: *A non opaque system w.r.t.* $\pi_{o,d}$

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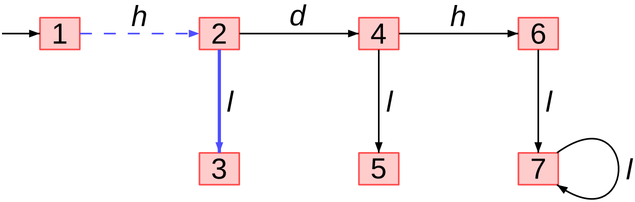

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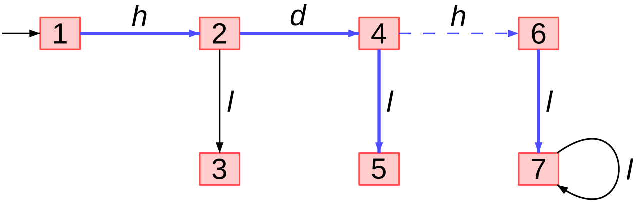

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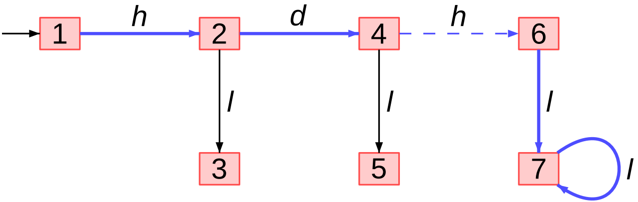

Figure: *A non opaque system w.r.t.* $\pi_{o,d}$

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Some additional facts and notations

- For $s \in L$, $[s]^L_\pi = \pi^{-1}(\pi(s))$
- $D(L)$= prefixes of L terminating with a d
- for $s \in D(L)$, $C(s, L)$ is its set of d-free continuations in L

Proposition (Factorization)

Any u in L admits a unique factorization $u = st$ *such that* $s \in D(L)$ *and* $t \in C(s, L)$.

- $[st]^L_{o,d} = s[t]^{C(s,L)}_o$

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Incorporating the secret into the transition system

Given

- $G = (\Sigma, Q, \delta, q_0, F)$ s.t. $L_F(G) = L$
- A regular secret $\varphi \subseteq L$

One constructs:

1. a complete deterministic transition system

$$G_\varphi = (\Sigma, Q', \delta', q'_0, F_\varphi) \text{ s.t. } L_{F_\varphi}(G_\varphi) = \varphi$$

2. $G_\# = G \times G_\varphi$, $F_\# = F \times Q'$ and $F_{\varphi\#} = F \times F_\varphi$.

Proposition

- $L_{F_{\varphi\#}}(G_\#) = \varphi$
- $L_{F_\#}(G_\#) = L$

Checking opacity w.r.t. an orwellian projection

Theorem

Let $Q_d = \{q : \cdot \xrightarrow{d} q\}$. φ is opaque for $L_F(G)$ w.r.t. $\pi_{o,d}$ iff for all $q \in Q_d$, $L_{F_\varphi}(G^q \setminus \Sigma_d)$ is opaque for $L_F(G^q \setminus \Sigma_d)$ w.r.t. π_o.

Proof.

$\Longleftarrow$: φ is not opaque for L w.r.t. $\pi_{o,d}$ then

$$(\exists s_0 \in D(\varphi), t_0 \in C(s_0, \varphi))\ [s_0 \cdot t_0]_{o,d}^{L_F(G)} \subseteq \varphi.$$

And as
$[s_0 \cdot t_0]_{o,d}^{L_F(G)} = s_0 \cdot [t_0]_o^{C(s_0, L_F(G))} = s_0 \cdot [t_0]_o^{L_F(G^{\overline{q}} \setminus \Sigma_d)}$ for $\delta(q_0, s_0) = \overline{q}$
We get that $[t_0]_o^{L_F(G^{\overline{q}} \setminus \Sigma_d)} \subseteq L_{F_\varphi}(G^{\overline{q}} \setminus \Sigma_d)$
and hence , $L_{F_\varphi}(G^{\overline{q}} \setminus \Sigma_d)$ is not opaque for $L_F(G^{\overline{q}} \setminus \Sigma_d)$ w.r.t. π_o. □

Outline

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Non-interference

Definition

Let a LTS $G = (\Sigma, Q, \delta, q_0, F)$ and $L = L_F(G)$ then L satisfies *non-interference* (NI) if $\pi_{Low}(L) \subseteq L$.

For $h \in High$
and $l \in Low$

1. Causal dependency on left handside: $h \Longrightarrow l$
2. No causal dependency on right hand side

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Intransitive Non-interference

Definition

Let a LTS $G = (\Sigma, Q, \delta, q_0, F)$ and $L = L_F(G)$ then L satisfies *intransitive non-interference* (INI) if $\pi_{High,Down}(L) \subseteq L$.

1. Not admissible causal dependency on left handside: $hh \Longrightarrow l$
2. Admissible causal dependency on right hand side

Theorem (Mullins2000)

Let a LTS $G = (\Sigma, Q, \delta, q_0, F)$ and $L = L_F(G)$. L satisfies INI iff for all $q \in Q_d$, $L_F(G^q \setminus Down)$ satisfies NI.

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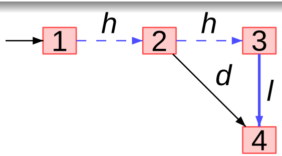

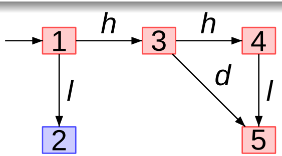

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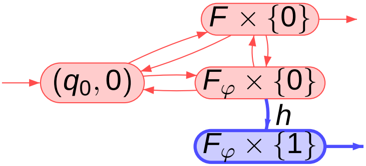
Opacity w.r.t. π_o reduces to NI

$G = (\Sigma, Q, \delta, q_0)$: LTS
$F, F_\varphi \subseteq Q$
$Low = \Sigma_o$, $High = \{h\}$



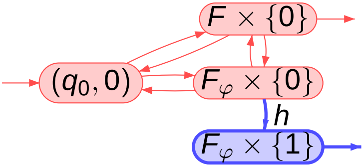
$G^\flat = (\Sigma^\flat, Q^\flat, \delta^\flat, q_0^\flat)$ and $F^\flat \subseteq Q^\flat$ s.t.

$$\begin{aligned}
\Sigma^\flat &= \Sigma_o \uplus \{h\} \\
Q^\flat &= (Q \times \{0\}) \cup (F_\varphi \times \{1\}) \\
\delta^\flat &= \{((q,0), \alpha, (q',0)) : (q, \alpha, q') \in \delta \text{ and } \alpha \in \Sigma_o\} \cup \\
&\quad \{((q,0), \epsilon, (q',0)) : (q, \alpha, q') \in \delta \text{ and } \alpha \in \Sigma_u\} \cup \\
&\quad \{((q,0), h, (q,1)) : q \in F_\varphi\} \\
q_0^\flat &= (q_0, 0) \\
F^\flat &= (F \cap (Q \setminus F_\varphi) \times \{0\}) \cup (F_\varphi \times \{1\}).
\end{aligned}$$

Opacity w.r.t. π_o reduces to NI (2)

The reduction

φ is opaque for L w.r.t. π_o iff $L^\flat = L_{F^\flat}(G^\flat)$ satisfies NI



Proof.

$\Longrightarrow$: Otherwise, there is $s'_0 \in \pi_{Low}(L^\flat)$ ($= L_{F_\varphi \times \{0\}}(G^\flat)$) s.t. $s'_0 \notin L^\flat$. Hence $s'_0 \in L_{F_\varphi \times \{0\}}(G^\flat)$ meaning that $[s_0]^L_o \subseteq \varphi$ for s_0 s.t. $\pi_o(s_0) = s'_0$ and consequently that s_0 discloses φ w.r.t. π_o.

$\Longleftarrow$: We show that if s_0 discloses the secret φ then let $s'_0 = \pi_o(s_0) \cdot h$ then $\pi_{Low}(s'_0) \notin L^\flat$ and $s'_0 \in L^\flat$ and hence, $L^\flat$ violates NI. □

Opacity w.r.t. $\pi_{o,d}$ reduces to INI

$G = (\Sigma, Q, \delta, q_0)$: LTS

$F, F_\varphi \subseteq Q$

$Low = \Sigma_o, Down = \Sigma_d$ and $High = \{h\}$

$G^\natural = (\Sigma^\natural, Q^\natural, \delta^\natural, q_0^\natural)$ and $F^\natural \subseteq Q'$ s.t.

The reduction

φ is opaque for L w.r.t. $\pi_{o,d}$ iff $L^\natural = L_{F^\natural}(G^\natural)$ satisfies INI

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INI reduces to opacity w.r.t. $\pi_{o,d}$

The construction

$G = (\Sigma, Q, \delta, q_0)$: LTS
$F \subseteq Q$ s.t. $L = L_F(G)$
$\Sigma = High \uplus Low \uplus Down$

$$\varphi = \{s \in L_F(G) : \pi_{Low,Down}(s) \neq s\}$$

The reduction

L satisfies INI iff φ is opaque for L w.r.t. $\pi_{Low,Down}$.

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Outline

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Summary

1. We have investigated
   - A class of orwellian observation functions
   - Computability of OVP in the context of
     - finite systems,
     - regular secrets and
     - orwellian projections
2. As an illustration of the relevancy to security we have related opacity w.r.t. $\pi_{o,d}$ to INI by
   - showing an equivalence between both notions,
   - providing, as a side effect a characterization of NI with opacity w.r.t. natural projections.

   also in the context of
   - finite systems,
   - regular secrets

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Future work

- Instantiate opacity to INI with selective downgrading (INISD), allowing to each downgrading action d to declassify only a subset $H(d)$ of Σ_u as defined in [Best, Darondeau (POST '12)].
- Instantiate to known information flow properties. (Non-deducibility on strategies) [Wittbold, Johnson (1990)],
- Address the problem of supremal sub-language L of K such that φ is opaque for L w.r.t. $\pi_{o,d}$ and its dual problem.
- Address the problem of supervisory control for opacity w.r.t. $\pi_{o,d}$ along the lines of [Dubreil, Darondeau, Marchand (2010)].

Thank you

Any questions ?